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Holographic Dark Energy in Induced Gravity

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Many astrophysics data show that our universe has a critical energy density, and 73% of it is dark energy, which drives the accelerating expansion of the universe. We consider the holographic dark energy in induced gravity by taking the Hubble scale, particle horizon and event horizon as the infrared cutoff. We find that only the event horizon can give accelerating expansion of our universe.

KEY WORDS: dark energy; cosmology; holography; induced gravity.

1. INTRODUCTION

pt Many astrophysical data show that our universe is almost flat, and currently undergoing a period of accelerating expansion (de Bernardis *et al.*, 2000; Netterfield *et al.*, 2002; Perlmutter *et al.*, 1999; Spergel *et al.*, in press; Tegmark *et al.*, 2004; Tonry *et al.*, 2003). It follows immediately thatour universe has a critical energy density. And about 73% of the energy density in our universe is dark energy which drives the accelerating. There are many candidates of dark energy such as the cosmological constant, quintessence (Caldwell *et al.*, 1998; Gao and Shen, 2002; Padmanabhan, 2003; Peebles and Ratra, 2003), phantom (Caldwell, 2002; Gu and Hwang, 2001; Sun and Shen, 2005) etc. Sometime ago Cohen, Kaplan and Nelson suggested that for any state in the Hilbert space with energy E, the corresponding Schwarzschild radius $R_s \sim E$ is less than the infrared (IR) cutoff L (Cohen *et al.*, 1999). Thus, the maximum entropy is $S_{BH}^{3/4}$. And a

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relationship between the ultraviolet (UV) cutoff and the infrared cutoff can be obtained, i.e. $8\pi GL^3\rho_\Lambda/3 \leq L$. The largest L allowed is the one saturating this inequality. Thus the holographic cosmological constant is

$$\rho_\Lambda = \frac{3}{8\pi GL^2} \tag{1.1}$$

The holographic cosmological constant ρ_Λ can play the role of dark energy in the present day by taking L as the size of the current universe. Hsu pointed that the holographic cosmological constant model based on the Hubble scale as IR cutoff won't give an accelerating universe (Hsu, 2004). Li found that the holographic dark energy model can give an accelerating universe by taking L as the event horizon Li (in press). Gong extend it to Brans–Dicke cosmology Gong (2004).

The concept of spontaneous symmetry breaking plays a very important role in modern particle physics. Zee suggested that it may also play a role in formulating a quantum theory of gravity (Zee, 1979). Their idea is to exclude the Einstein term from the definite action, and have it induced in the effective action. This theory behaves like general relativity at low energy, but show different at very high energy. Some cosmology topics have been discussed in induced gravity (Accetta *et al.*, 1985; Shen, 1995). In this paper,we discuss the holographic dark energy with induced gravity.

2. EVOLUTION EQUATIONS OF OUR UNIVERSE

The action of induced gravity is

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}\epsilon\varphi^2 R - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi) + L_w\right] \tag{2.1}$$

Here φ represents a scalar field and ϵ is a dimensionless coupling constant that is of order $\lesssim 1$. L_w is the Lagrangian for the rest of the universe and does not include φ in our paper. $V(\varphi)$ is minimized when $\varphi = v$. Here v is the vacuum expectation value of φ. We take the following form of $V(\varphi)$ in our paper

$$V(\varphi) = \frac{1}{8}\lambda(\varphi^2 - v^2)^2 \tag{2.2}$$

where λ is a dimensionless coupling constant.

The effective value of Newton's constant

$$G_{\text{eff}} = \frac{1}{16\pi}\left(\frac{1}{\frac{1}{2}\epsilon\varphi^2}\right) \tag{2.3}$$

It becomes G_N when $\varphi = v$ where v is the vacuum expectation value. We can see that (2.1) would reduce to Einstein's action by setting $\varphi = v$. In present day, $\varphi(x) = v$. Thus, $V(\varphi) = \frac{1}{8}\lambda(\varphi^2 - v^2)^2 = 0$, and we can withdraw it in the following.

In our case, we use the flat Robertson–Walker metric which is given by

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2) \tag{2.4}$$

where $a\,(t)$ is the scale factor of the universe.

The metric (2.4) has the following non-trivial Christoffel symbols

$$\Gamma^1_{10} = \Gamma^2_{20} = \Gamma^3_{30} = \frac{\dot{a}}{a},$$
$$\Gamma^0_{11} = \Gamma^0_{22} = \Gamma^0_{33} = a\dot{a}. \tag{2.5}$$

The nontrivial components of the Ricci tensors are

$$R^0_0 = -3\left(\frac{\ddot{a}}{a}\right), \tag{2.6}$$

$$R^1_1 = R^2_2 = R^3_3 = -\left(\frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2}\right). \tag{2.7}$$

From above, we can obtain the Ricci scalar

$$R = -6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right). \tag{2.8}$$

The evolution equations of the universe derived from the action (2.1) are

$$3H^2\epsilon\varphi^2 = \rho + \frac{1}{2}\dot{\varphi}^2 + V(\varphi) - 6H\epsilon\varphi\dot{\varphi} \tag{2.9}$$

$$\varphi\ddot{\varphi} + \dot{\varphi}^2 + 3H\varphi\dot{\varphi} = \frac{1}{1+6\epsilon}(\rho - 3P) \tag{2.10}$$

$$\dot{\rho} + 3H(\rho + P) = 0 \tag{2.11}$$

where ρ and P is the energy density and pressure of the perfect fluid $T^{\mu\nu} = (\rho + P)U^\mu U^\nu - Pg^{\mu\nu}$.

3. ACCELERATING EXPANSION

The universe is dark energy dominated in the present day, so we set $\rho_\Lambda = \rho$. First, we take $L = H^{-1}$. Substituting the relation into Eq. (1.1), we can get

$$\rho = 3H^2\epsilon\varphi^2 \tag{3.12}$$

and then

$$\dot{\rho} = 6H^2\epsilon\varphi\dot{\varphi} + 6H\epsilon\varphi^2\left(\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}\right) \tag{3.13}$$

Substituting Eqs. (3.12) and (3.13) into the evolution equations, we can get power-law solutions:

$$a(t) = a_0 t^{\alpha} \tag{3.14}$$

$$\varphi(t) = \varphi_0 t^{\beta} \tag{3.15}$$

where

$$\alpha = \frac{1}{4(1+6\epsilon)} \tag{3.16}$$

$$\beta = \frac{3\epsilon}{1+6\epsilon} \tag{3.17}$$

An accelerating universe requires $\alpha > 1$, then $\epsilon < -\frac{1}{8}$. It is impossible, because of $G_N = \frac{1}{16\pi}(\frac{1}{\frac{1}{2}\epsilon v^2}) > 0$. Thus the choice of Hubble scale as the IR cutoff can not give an accelerating universe. Fischler and Susskind have proposed taking the particle horizon R_H as the IR cutoff (Fischler and Susskind, in press). The particle horizon is

$$R_H = a\int_0^t \frac{dt}{a} = \frac{t}{1-\alpha} \tag{3.18}$$

by assuming power-law solution (3.16) and (3.15). Taking $L = R_H = \frac{t}{1-\alpha}$ in our model, we have

$$\rho = \frac{3\epsilon\varphi^2(1-\alpha)^2}{t^2} \tag{3.19}$$

$$\dot{\rho} = \frac{6\epsilon\varphi\dot{\varphi}(1-\alpha^2)}{t^2} - \frac{6\epsilon\varphi^2(1-\alpha)^2}{t^3} \tag{3.20}$$

Substituting Eqs. (3.19) and (3.20) into the evolution equations, we get

$$\frac{6(1-\alpha)^2(1-\beta)}{\alpha} + \frac{1+6\epsilon}{\epsilon}(3\alpha\beta + 2\beta^2 - \beta) = 12(1-\alpha)^2 \tag{3.21}$$

$$3 - 6\alpha + \frac{\beta^2}{2\epsilon} - 6\alpha\beta = 0 \tag{3.22}$$

Solving Eqs. (3.21) and (3.22), we obtain

$$\alpha = \frac{1}{2} \tag{3.23}$$

$$\beta = 0 \tag{3.24}$$

Thus, the choice of particle horizon as the IR cutoff does not give an accelerating expansion too. In fact this result is expected, because induced gravity becomes general relativity when $\beta = 0$. Finally, we take the event horizon R_h as the IR cutoff.

$$L = R_h = a \int_t^{\infty} \frac{dt}{a} \tag{3.25}$$

Thus

$$\rho = \frac{3\epsilon\varphi^2}{R_h^2} \tag{3.26}$$

Substituting (3.26) into Eq. (2.9) and assuming $\frac{\varphi}{\varphi_0} = (\frac{a}{a_0})^m$, we get

$$R_h H = \frac{1}{n} \tag{3.27}$$

where $n = \sqrt{1 - \frac{m^2}{6\epsilon} + 2m}$. From Eq. (3.27), we can get

$$\frac{H}{H_0} = \left(\frac{a}{a_0}\right)^{n-1} \tag{3.28}$$

Thus

$$\rho = 3\epsilon\varphi_0^2 H_0^2 n^2 \left(\frac{a}{a_0}\right)^{2n+2m-2} \tag{3.29}$$

$$\dot{\rho} = 3\epsilon\varphi_0^2 H_0^2 n^2 \left(\frac{a}{a_0}\right)^{2n+2m-2} H(2n + 2m - 2) \tag{3.30}$$

Substituting (3.29) and (3.30) to the evolution equations, we get

$$\frac{1+6\epsilon}{\epsilon} m(2m + n + 2) = 3n^2(2n + 2m + 2) \tag{3.31}$$

Then we can get

$$m = -\frac{6^{\frac{2}{3}}\epsilon}{(6\epsilon^2 + \sqrt{6}\sqrt{\epsilon^3 + 6\epsilon^4})^{\frac{1}{3}}} + 6^{\frac{1}{3}}(6\epsilon^2 + \sqrt{6}\sqrt{\epsilon^3 + 6\epsilon^4})^{\frac{1}{3}} \tag{3.32}$$

and

$$n = \sqrt{1 + 2\left(-\frac{6^{2/3}\epsilon}{d} + 6^{1/3}d\right) - \frac{\left(-\frac{6^{2/3}\epsilon}{d} + 6^{1/3}d\right)^2}{6\epsilon}}, \tag{3.33}$$

where

$$d = (6\epsilon^2 + \sqrt{6}\sqrt{\epsilon^3 + 6\epsilon^4})^{1/3}. \tag{3.34}$$

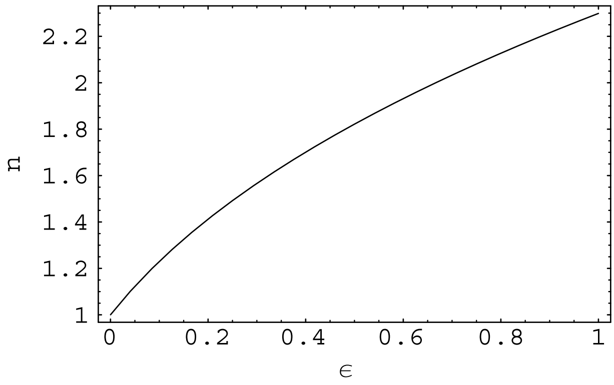

Fig. 1. The value of n depending on ϵ.

The value of n depending on ϵ is shown in (Fig. 1). Therefore, the expansion of the universe is accelerating.

4. CONCLUSIONS

We show that in induced gravity the Hubble scale and the particle horizon do not provide the holographic dark energy, but the event horizon gives the holographic dark energy which drives the accelerating expansion of our universe. In fact this result is expected. Because induced gravity becomes general relativity in the present day. And we know that in standard cosmology there is a similar conclusion. Thus, taking the event horizon as the IR cutoff should provide the holographic dark energy in induced gravity.

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